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The Mechanism of the Electric Arc

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V. *The Mechanism of the Electric Arc.*

By (*Mrs.*) HERTHA AYRTON.

Communicated by Professor J. PERRY, *F.R.S.*

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EVER since the first discovery of the Electric Arc, nearly 100 years ago, the secret of its mechanism has been one of the most fascinating mysteries of science. To account for its abnormally high temperature, and for the fact that a higher P.D. is required to send a small current than a large one through it, the arc has been endowed with unique properties, such as a back E.M.F. of many volts, and even a negative resistance. The measurement of this resistance alone has been the object of a large number of experiments, made under all conceivable conditions.

The object of the present paper is to see how far this peculiar behaviour of the arc might have been logically predicted from the known conditions of its existence, viz., that it is a gap in a circuit, furnishing its own conductor by the evaporation of its own material; and to show that it is quite unnecessary to invoke the aid of a negative resistance, or even of a large back E.M.F., to account for this behaviour.

What happens on making the Gap.

The usual explanation given for the formation of a spark or flash, on opening an electric circuit, is that it is caused by self-induction. The interesting question therefore arises, could an arc be struck and maintained if there were no self-induction whatever in the circuit? I think it could. For the surfaces of all solids are irregular, and therefore all parts of the carbons cannot be separated at the same instant. The parts that remain in contact will still conduct the current, but the fewer of them that remain the greater will be their resistance. The heat caused by this resistance must, at last, be great enough to volatilise the carbon at the remaining points of contact, and, by the time that no part of one carbon is touching any part of the other, the small gap will be full of carbon vapour. [As the carbon points at each junction must volatilise as soon as they are hot enough to do so, this vapour will be given off at a constant temperature, viz., the lowest at which carbon can volatilise. —March 23, 1902.]

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To explain the further formation of the arc, we must remember that when the carbons are separated still more all the material in the gap cannot retain its high temperature. The access of the cold air must, I consider, turn some of the vapour into *carbon mist* or *fog* as distinct from *carbon vapour*, just as the steam issuing from a kettle is turned into visible mist at a short distance from its mouth. The interior globular portion A (fig. 1), which is purple in the image of the arc, is, I suggest, composed of such carbon mist, while there is an indication of a space between

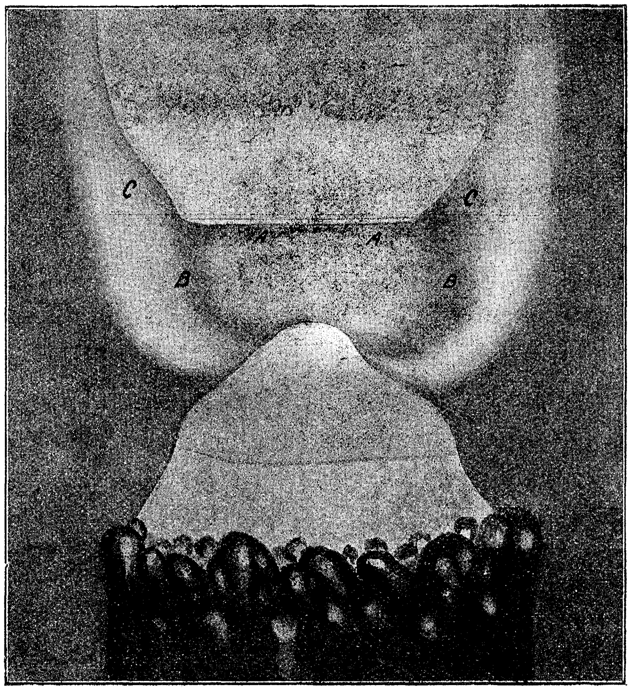

Fig. 1. Enlarged image of arc and carbons with positive carbon on top. AA, purple mist, BB, shadow, CC, green flame.

this mist and the positive carbon which is occupied, I believe, by a thin film of true carbon vapour.

Next the dissimilar action of the poles, met with in so many electric phenomena, begins. Instead of *both* poles volatilising, so that there is a thin layer of carbon *vapour* over each with a mass of carbon *mist* between them, the positive pole alone volatilises, while the negative appears simply to burn away.

Besides the film of vapour and the bulb of mist, other volatile materials must go to make up the whole substance of the arc. For the surrounding air must not only cool the carbon vapour, but it must unite chemically with a certain thickness of the mist, thus forming a sheath of burning gases surrounding both vapour and mist, and even portions of the solid carbons themselves. This sheath of gases, which is of a brilliant green colour with solid carbons, may be seen at C (fig. 1), while B, the shadow between it and the mist, probably indicates where the two mingle. There

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must be three sorts of material in the gap, therefore, marking the three stages through which the vapour is continually passing.

1. It starts as a thin film of carbon *vapour* spread over the end of the positive carbon.
2. It then changes into the *mist* that lies between this vapour film and the negative carbon.
3. Finally it burns and forms a sheath of burning gases which encloses not only the fresh vapour and mist, but also the ends of the solid carbons themselves.

The Conducting Power of the Vapour, Mist, and Flame.

The specific resistances of true vapours have been shown to be high, therefore I conclude that the film over the end of the positive carbon has a high resistance, even though it be very thin. The mist, on the contrary, is composed, I think, of minute solid particles of carbon, and must, therefore, I anticipate, have a lower specific resistance. My experiments on the flame have shown, on the other hand, that its specific resistance is so high, compared with that of the inner purple mist, that it is relatively an insulator, a result confirming that obtained by LUGGIN* in 1889. The current, therefore, flows through the vapour and the mist, and practically not at all through the sheath of burning gases.

The Production of the High Temperature at the Crater.

To explain the great production of heat at the end of the positive carbon, as well as the sudden change of potential that is known to exist there, it has been supposed that a back E.M.F. of some 35 to 40 volts existed at the junction of the crater and the arc. But if, as I suggest, there be a high resisting vapour film in contact with the crater, the current passing through this must generate much heat, and this heat is utilised mainly in continuously forming fresh carbon vapour, at the lowest temperature at which carbon will volatilise—to be itself turned into mist, and then into flame. Hence it seems probable that the high and constant temperature of the crater is kept up by the current flowing, not against a back E.M.F., but through the resistance of a thin vapour film of constant temperature lying over the surface of the crater. In other words, *it is not the crater itself that is the source of the heat of the arc, but a thin film of carbon vapour, at constant temperature, in intimate contact with it.*

* 'Wien. Sitzungsberichte,' vol. 98, p. 1, 233.

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Why the End of the Positive Carbon has its Particular Shape.

As only the part of the positive carbon that is in actual contact with the vapour film can be at the temperature of volatilisation, evaporation can only take place at that surface, and hence I suggest that, unless the vapour film is as large as the whole cross-section of the positive carbon, it must dig down into the carbon and leave the surrounding parts unvolatilised, *i.e.*, the part of the positive carbon against which the film rests must become concave. These surrounding parts, however, are heated sufficiently by conduction from the evaporating surface and by the hot gases surrounding them to burn away, and so there must be a race between volatilisation of the centre portion and burning away of the edges, which must, in all cases, determine the shape of the surface of volatilisation. When, all other things being equal, the gap between the carbons is small, so that the end surface of each carbon is well protected from the air, volatilisation will gain over burning and the pit may become very deep. When, on the other hand, the gap is large, so that the air can easily reach all parts of the carbon except that actually covered by vapour, these parts may burn away as fast as, or even faster than the inner portion is volatilised, and in that case the surface of volatilisation will be flat, or even slightly convex. It is evident, therefore, that this surface cannot, from the very nature of things, help being concave when the distance between the carbons is short, and flat or convex when it is long. And this is true, whether the volatilisation is due solely to a large back E.M.F., as some have supposed, or to the resistance of a thin film of carbon vapour, as I have suggested, or partly to one and partly to the other.

When only a small bit of the end of the positive carbon is being volatilised, the outer edge of the carbon will *not* be made hot enough to burn, and the tip will remain relatively blunt. When, on the contrary, the area of volatilisation is large, the edge of the carbon will be burnt away and a long tapering end will be formed, terminating in the surface of volatilisation. Further, the shorter the arc, the less easily will the heat be able to escape from between the carbons, so that the more remains in them to produce burning, and, consequently, the longer must be the tapering part. Experience shows these conclusions to be true.

Why the End of the Negative Carbon assumes its Particular Shape.

The negative carbon is shaped entirely by burning away; the heat that raises it to burning temperature being furnished partly by the mist that touches it, and partly by radiation from the vapour film lying against the positive carbon. The part that the mist rests on is protected by it from the action of the air, and does not, therefore, burn away. At the same time this part must be hotter than the remainder of the carbon, and so the portion of the carbon near it burns away more readily than the

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rest, leaving a mist-covered tip which is longer and slenderer, because its sides are hotter and burn away more readily, the larger the crater and the shorter the arc.

Hence, with a small crater and a long arc, the negative carbon remains fairly flat, as in *a*, (fig. 2); whereas, as the crater becomes larger, *its* action alone shapes the negative carbon as dotted in *b*, (fig. 2), and the extra heating due to the mist combined with the protection which the mist offers as shown in the full line. With a short arc, on the contrary, a small crater alone would produce an end as dotted in *c*, (fig. 2), while the combined effect of the crater and mist produce the end outlined by the full line. Finally, with a large crater and a short arc, the crater alone would produce an end as dotted in *d*, (fig. 2), while the crater and mist together would shape the negative carbon as given by the full line in *d*. Experience shows that the negative carbon does shape itself in this way under the various conditions.

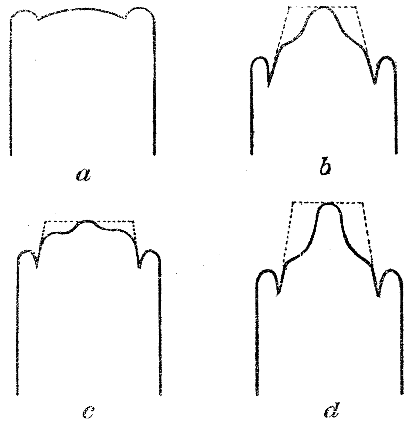

Fig. 2. Negative carbons. *a*, long arc, small crater; *b*, long arc, large crater; *c*, short arc, small crater; *d*, short arc, large crater.

Why the Area of the Crater is not Directly Proportional to the Current, but Depends also on the Length of the Arc.

Suppose that the current and the distance between the ends of the carbons have been kept constant long enough for all the conditions of the arc to have become steady, so that it is "*normal*," and that then the resistance in the outside circuit is suddenly diminished. At the first instant the P.D. between the carbons must be increased, a larger current will have to flow through a vapour film of the old dimensions, and consequently the heat developed in it per second will increase. The temperature of the existing vapour film cannot rise, because there is no increase of pressure, consequently it must expand, and spread over a larger area of solid carbon. The moment the film had expanded in the slightest degree, it would begin volatilising carbon from a part of the surface hitherto inactive, and thus a larger quantity of vapour per second would be volatilised. At the next instant, therefore, the quantity of carbon volatilised per second would have increased, and the resistance of the vapour film would have become lower, and its tendency to expand would, therefore, be diminished on both accounts. Thus, at each instant after the change of current the volatilising surface would increase, but more and more slowly, till its area was such that the heat developed per second in the vapour film only just sufficed, after all losses from conduction, &c., to keep up the volatilisation. After that, the vapour film would cease to expand, and the surface of volatilisation would have reached its maximum area for the new current.

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The vapour film, besides radiating heat in all directions from its free surface, must lose a certain extra amount of heat all round its edges by conduction through a ring of the solid carbon that it does not actually touch. The heat thus lost must be subtracted from the edges of the part of the solid carbon that the vapour film does touch, and this part will, therefore, be just *below* the temperature of volatilisation, as will also the small ring of solid carbon outside the vapour film. Suppose, for instance, that the full line in fig. 3 is the part of the positive carbon that is in contact with the vapour film, then the inner dotted line will enclose the area that is actually volatilising fresh carbon, and the space between the two dotted circles will be at a temperature just below that of volatilisation, because the conduction of heat from the edges of the vapour film will bring the outer circle up and the inner circle down to a temperature a little below that of the vapour film itself. The slightly lower temperature of the space between the dotted circles would make it perhaps a little less brilliant than the volatilising surface, but it would still be very much more brilliant than the remainder of the positive carbon, so that it must form the outer circle of what we are accustomed to call the crater, viz., the most brilliantly white part of that carbon. The area of the crater is thus rather larger than the cross-section of the vapour film, while the actively volatilising surface is slightly smaller.

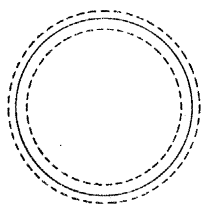

Fig. 3.

When the carbon vapour proceeds from a given area, the cross-section of the vapour film will be greater the more it is protected from the cold outer air by the end of the positive carbon. If, for instance, AB, fig. 4, were the diameter of the

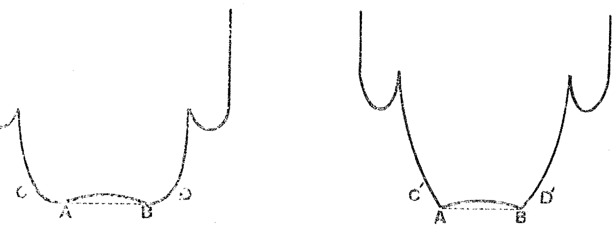

Fig. 4. Positive carbons having the same area of volatilisation. CABD with a long arc. C′ABD′ with a short arc.

volatilising surface, the cross-section of the vapour film would be greater if the end of the carbon were CD, than if it were C′D′, or, since the end of the positive carbon is thicker the longer the arc, *the cross-section of the vapour film is greater the longer the arc.* This film will also be able to keep a larger ring of solid carbon at a temperature just below the lowest at which volatilisation can take place, when the end of the carbon is CD, than when it is C′D′, therefore the whole space that is just below the lowest temperature of volatilisation, *i.e.*, that included between the dotted circles in fig. 3, will be greater with a long arc than with a short one, when the

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surface of volatilisation is the same in each case. In other words, the area of the crater increases with the length of the arc with a given surface of volatilisation. Now, I shall show presently that, in the normal state of the arc, the area of the volatilising surface is directly proportional to the current, but is independent of the length of the arc; it follows, therefore, that with a given constant current the area of the *crater increases* with the length of the arc, as I have found it to do by actual measurement.*

The area of the crater, then, if we define it as that part of the positive carbon that is far brighter than the rest, is not a function of the current only, as has hitherto been affirmed. It is a function of the current, the length of the arc, and, until the arc has become normal after any changes have occurred in the length or the current, of the time after the change was made. The cross-section of the vapour film, on the other hand, *is* proportional to the current, as we shall now see.

The Film of Vapour in Contact with the Positive Carbon acts like a Back E.M.F.

Let a be that area of the film that uses its heat in volatilising fresh carbon, and let x be the part of which the heat is lost by conduction, radiation, &c. Then the whole area of the film is $a + x$, and its resistance, if we consider the thickness of the film to be constant, is $\frac{p}{a+x}$, where p is a constant. The heat generated per second in the film varies as $\frac{pA^2}{a+x}$ and, of this, only $\frac{a}{a+x}$ is used in volatilisation.

The quantity of carbon volatilised per second is, therefore, proportional to

$$\frac{a}{a+x} \cdot \frac{pA^2}{a+x}, \text{ or } \frac{apA^2}{(a+x)^2}.$$

But, therefore, since the temperature at which volatilisation is taking place is a constant one, viz., the lowest possible, the quantity of carbon volatilised per second must be proportional to the area of the surface from which it is volatilised, *i.e.*, to a.

$$qa = \frac{paA^2}{(a+x)^2}, \text{ where } q \text{ is constant,}$$

That is, $a + x$, the area of the vapour film, is proportional to A, the current.

Again, from the above,

$$\frac{a+x}{p} = \frac{A^2}{(a+x)\,q};$$

but

$$\frac{a+x}{p} = \frac{1}{f}, \text{ where } f \text{ is the resistance of the film,}$$

therefore

$$f = \frac{q\,(a+x)}{A^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

* 'The Electric Arc,' p. 154.

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Or, since $a + x$ is proportional to A,

$$f = \frac{k}{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

where k is constant; that is, Af, the P.D. used in sending the current through the vapour film, is constant.

Hence, no back E.M.F. at the crater is necessary to account for the great fall of potential between it and the arc, for the film of high resistance vapour, whose existence I have suggested, could cause the P.D. between the positive carbon and the arc to remain constant, exactly as if this junction were the seat of a back E.M.F.

The Apparent Negative Resistance of the Arc is caused by the True Positive Resistance diminishing more rapidly than the Current Increases.

It has been mentioned (p. 301) that the specific resistance of the green flame is so high as to make it, to all intents and purposes, an insulator, so that nearly the whole of the current flows through the mist. It follows, therefore, that the resistance of an arc of given length must depend (apart from the resistance of the vapour film) simply on the cross-section of the carbon mist, which, as it appears purple in the image of the arc, can easily be measured. To see how this cross-section varies when the current is increased while the length of the arc is kept constant, I have drawn, in fig. 5, diagrams traced from actual images, after the arc had been burning long enough with each current and length for the P.D. between the carbons to have become quite constant, great care having been taken to trace as accurately as possible the exact limits of the purple centre and the green outer flame.

The resistance of the carbon mist (as distinct from that of the vapour film) may be defined, practically, as being the resistance of that portion of the mist that lies between the parallel planes passing through the mouth of the crater and the tip of the negative carbon.

The mean cross-section of the mist D^2, given in column 3 of Table I., has been obtained by taking the means of the squares of the three lengths AB, CD, and EF. The next column, giving the ratio of D^2 to the current A, shows that the cross-section of the mist increases more rapidly than the current. Column 5 gives numbers proportional to the resistance of the mist, while columns 6 and 7 contain numbers proportional to the power spent in the mist, as obtained from these experiments and from the equation to be subsequently referred to.

The mist carries practically the whole of the current, and, since D^2 increases more rapidly than A (column 4), it follows that in the normal arc the resistance of the mist diminishes more rapidly than the current increases. But equation (2) above shows that the resistance of the vapour film varies inversely as the current. Hence, *with*

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solid carbons, the whole resistance of the normal arc diminishes more rapidly than the current increases, and consequently the P.D. must diminish as the current increases.

Thus, if, in the normal arc, δV be a change made in V, and δA be the corresponding

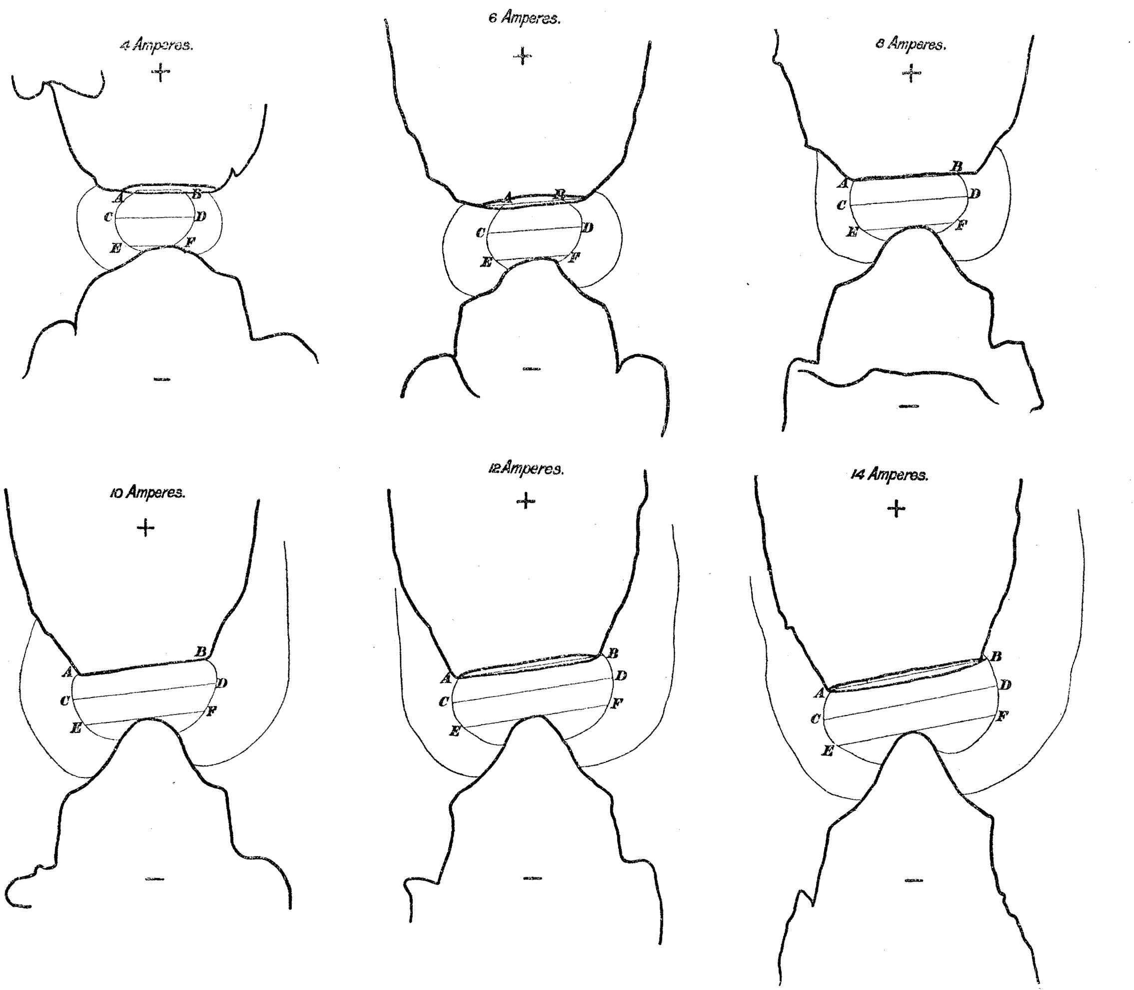

Fig. 5. Diagram of 2-millim. normal arc between solid carbons, positive 18 millims. and negative 15 millims. in diameter.

change produced in the current, $\frac{\delta V}{\delta A}$, has a negative value, even although the resistance of the arc is positive, simply because that resistance diminishes faster than the current increases, and *vice versâ*.

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TABLE I.—Mean Squares of Diameters of Mist with Corresponding Currents A, and Potential Differences V, between the Carbons. Numbers proportional to Resistances of Mist (Column 5), and Powers Expended in Mist (Column 6). Normal Arc.

Constant Length of Arc = 2 millims.

Solid Carbons, Positive 11 millims., and Negative 9 millims., in diameter.

1.	2.	3.	4.	5.	6.	7.
A.	V.	D^2.	$\frac{D^2}{A}$.	$\frac{1}{D^2}$.	$\frac{A^2}{D^2}$. From Experiment.	$\frac{A^2}{D^2}$. From Equation.
4	51·7	4·8	1·20	0·208	3·33	3·4
6	49·0	9·8	1·63	0·102	3·67	3·68
8	48·0	16·2	2·02	0·061	3·95	3·95
10	47·0	23·4	2·34	0·043	4·27	4·22
12	45·7	34·9	2·91	0·029	4·13	4·49
14	45·1	41·2	2·94	0·024	4·76	4·76

There is Nothing to show that the P.D. between the Carbons Divided by the Current is not the True Resistance of the Arc.

Fig. 6 shows that the curve connecting the values of A^2/D^2 given in the sixth column of Table I., with those of the current given in the first column, is a straight line having the equation

$$A^2/D^2 = 0{\cdot}136\ A + 2{\cdot}86.$$

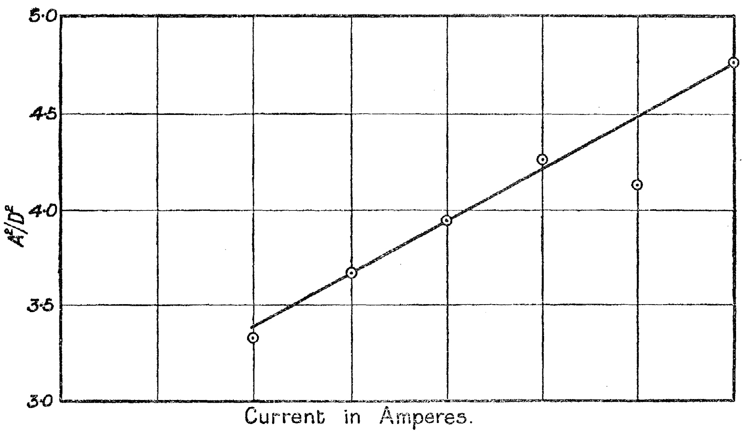

Fig. 6. Curve connecting the power expended in the arc mist with the current. Solid carbons, 11 millims. and 9 millims. in diameter. Length of arc, 2 millims.

Hence, for a normal arc of given length, the power expended in the carbon mist is

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proportional to a constant plus a term which varies directly with the current. Dividing by A^2, we obtain m, the resistance of the *mist*,

$$m = \frac{0{\cdot}136}{A} + \frac{2{\cdot}86}{A^2}.$$

Combining this with f, the resistance of the vapour film on p. 306, we have for the total resistance of the normal arc an expression of the form

$$f + m = \frac{\alpha}{A} + \frac{\beta}{A^2}.$$

But I have shown* that an equation of the form

$$V = a + bl + \frac{c + dl}{A}$$

exactly fits not only all the numerous measurements that I have myself made of simultaneous values of the P.D. between the carbons, the current, and the length of the arc, but also all the similar experiments made by other observers when both carbons are solid. When l, the length of the arc, is constant, this equation becomes

$$V = \gamma + \frac{\delta}{A},$$

where γ and δ are constants. Hence, dividing by A, we have, for the total apparent resistance of an arc,

$$r = \frac{\gamma}{A} + \frac{\delta}{A^2}.$$

Thus, by considering only the way in which the resistances of the vapour film and of the carbon mist respectively must vary with the current, we arrive at an equation for the resistance of exactly the same form as is obtained by dividing by the current the values found experimentally for the P.D. between the carbons. So that instead of an arc consisting of a circuit of low resistance combined with a back E.M.F., it may well be that its *apparent* resistance, *i.e.*, the ratio of V to A, is its *true* resistance; or it may be that, if there is any back E.M.F. at all, it is very much smaller than has hitherto been supposed.

Both the Resistance of the Arc and the P.D. between the Carbons depend not only on the Current and the Length, but also on How Lately a Change has been made in Either and on What that Change was.

The whole resistance of the arc depends on the cross-sections of the vapour film and the mist, and on the distance between the carbons. Now I have shown that

* 'The Electric Arc,' p. 186.

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when the P.D. between the carbons is changed—increased, say—the *first* result must be an increase of current, while the *second* is a corresponding increase in the cross-sections of the vapour film and the mist, causing a *diminution* of the resistance, and, consequently, of the P.D. between the carbons. *Thirdly*, if the new current is kept constant long enough, the end of the negative carbon burns away to a longer slenderer point, allowing more of the mist to escape, so that it takes a smaller cross-section, and, consequently, both the resistance and the P.D. increase again, although they never reach such high values as they had with the smaller current.

Fig. 7 is useful as showing at a glance how the resistance and the P.D. depend upon the time that has elapsed after a change of current. When the arc is normal,

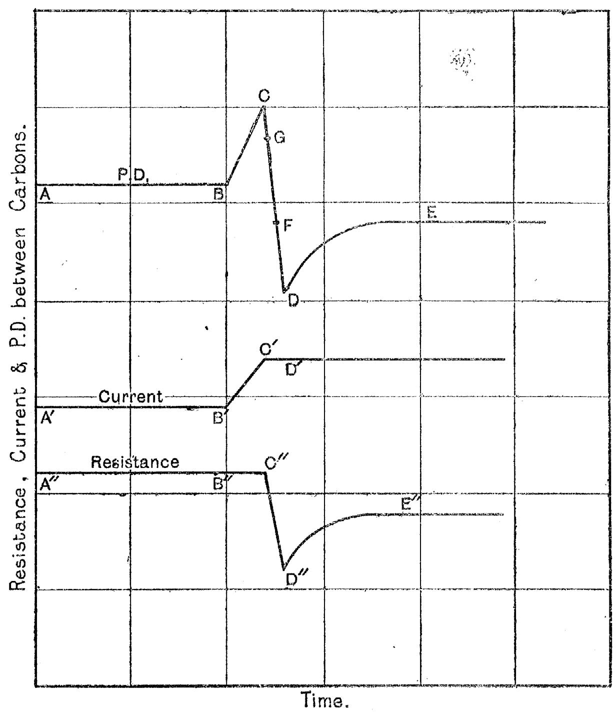

Fig. 7. Suggested simultaneous time-changes of P.D., current and resistance.

in the first instance, A B, A′ B′, and A″ B″ represent the curves connecting the P.D., the current, and the true resistance of the arc respectively with the time. When the P.D. is increased from B to C, the resistance does not alter at the first instant, but the current rises to C′. If it is then kept constant at C′, the surface of volatilisation next increases in area, the resistance falls to D″, and the P.D. consequently falls to D. After this the carbons begin to grow longer points; the cross-section of the mist diminishes, the resistance, therefore, increases to E″, and the P.D. with it to E. The arc has now become normal again, so that the curves are all now parallel straight lines, the current higher than before, and the P.D. and resistance lower.

Thus any alteration that is made and maintained in the arc sets up a series of changes in its resistance and, consequently, in the P.D. between the carbons, that

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cease only when the arc becomes normal again. In other words, when an arc of given length, with a given current flowing, exists between given carbons, neither the resistance nor the P.D. between the carbons has any fixed value, except when the arc is, and continues to be normal. In all other cases each varies, within certain limits, according to the time that has elapsed since either the current, or the length was altered, and according to what change was then made.

That the P.D. does actually undergo alterations of the kind just described, after a change of current, is evident from fig. 8, the curves in which were plotted from actual experiments made in 1893. For these curves, the current was suddenly altered when the arc was quite normal, and was then kept constant at the new value while the P.D. continued to alter, the time change of P.D. being noted. The first change of P.D.—the *rise* or *fall* that must have instantaneously accompanied the

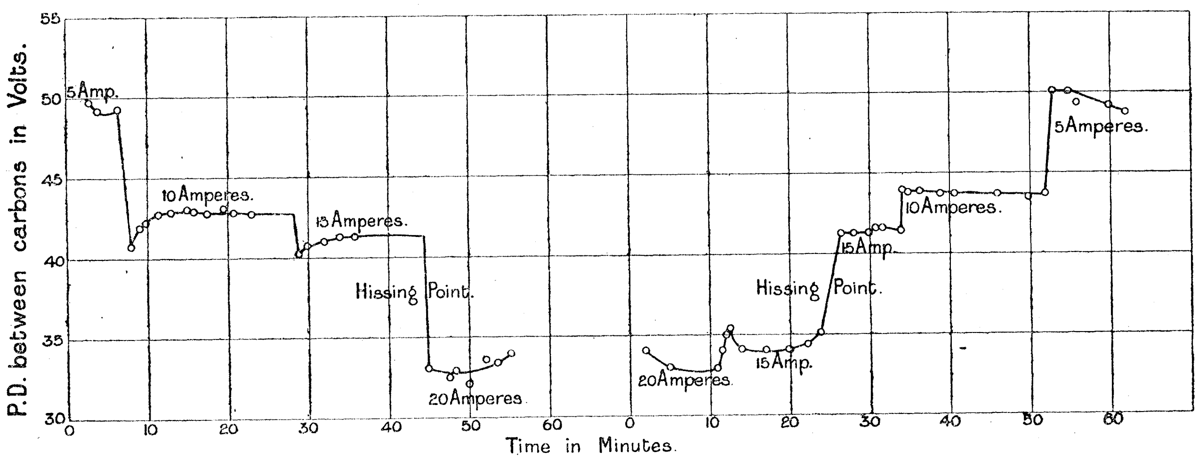

Fig. 8. Experimental curves showing simultaneous time-changes of P.D. and current with a constant length of arc of 1 millim. Solid carbons, 18 millims. and 15 millims. in diameter.

change of current, when the resistance outside the arc was suddenly diminished—was too quick to detect. Indeed, it was only after assisting in carrying out these experiments that it occurred to me that we ought to have seen it, and that, on trying, I found I could sometimes detect it and sometimes not.*

The rapid change in the P.D. while the area of the crater and the cross-section of the mist were altering, is very marked, however, as well as the slow rise or fall of P.D. accompanying the diminution or increase in the cross-section of the arc due to the change in the cross-section of the carbon ends.

Why Measurements of the Resistance of the Arc made under the same Apparent Conditions Differ in Value and even in Sign with Different Experimenters.

The dependence of the resistance of the arc on its previous history, as well as on the actually existent length and current, has an important bearing on the question

* It is only when the carbons are cored that it can be detected. The reason will be explained later.

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of measuring the resistance by means of a small superimposed alternating current. Such a method has been employed by many experimenters, but the results have not shared the similarity of the methods; for while VON LANG and ARONS found, in 1887, that the arc had a *positive* resistance, Messrs. FRITH and RODGERS, in 1896, found that it had a *negative* one with solid carbons.

We shall now see the reason of this disparity, and first it may be well to recall shortly the reasoning on which the method is based.

The equation $V = E + Ar$ may be taken to represent the connection between the P.D. between the carbons, the current, and the length of the arc, whether it has a variable E.M.F., a constant E.M.F., or none at all. For in the first case E will be variable, in the second constant, and in the third zero. In any case $\delta V/\delta A = r$, *only when such a small quick change is made in V and A that neither E nor r is made to vary by it.*

Instead of a single small quick change of current, the experimenters superimposed a small alternating current on the direct current of the arc, and measured the *average* value of $\delta V/\delta A$, or its equivalent. Obviously, if the alternating current left the resistance and any back E.M.F. that might exist in the arc unaffected, this was a *true* measure of the resistance of the arc. But if the alternating current changed both or either of these, then instead of being equal to r, we should have

$$\frac{\delta V}{\delta A} = r + \frac{\delta E}{\delta A} + A\frac{\delta r}{\delta A}$$ . . if there is a back E.M.F., and if both it and the resistance varied with the alternating current;

$$\frac{\delta V}{\delta A} = r + \frac{\delta E}{\delta A}$$ . . . . if there is a back E.M.F., and if it alone varied;

$$\frac{\delta V}{\delta A} = r + A\frac{\delta r}{\delta A}$$ . . . . if there is no back E.M.F., or if the resistance alone varied.

None of the experimenters, as far as I am aware, applied any but a few imperfect tests to see whether the alternating currents they employed affected the resistance of the arc or not, and it was, I believe, because the resistance *was* affected, in every case, that such diverse results were obtained. The low frequency of the alternations was the probable source of error, for I shall now show that, with a given root mean square value of the alternating current, the average value of $\delta V/\delta A$ varies not only in magnitude, but even in sign, with the frequency of that current.

Effect of the Frequency of the Superimposed Alternating Current on the Value and Sign of $\delta V/\delta A$.

I have shown (p. 310) that when a sudden increase of the current is made and maintained the P.D. has three successive stages of variation. It first rises (BC, fig. 7), then falls (CD), then rises again (DE), but not so high as before, and after this it

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remains constant. In dealing with a superimposed alternating current there is, of course, no sudden increase and diminution, everything is gradual. The three changes of P.D. do not, therefore, act separately—they overlap. At any moment, for instance, when the current is increasing say, the increase may be considered due to the addition of successive small increments of current, so that the P.D. has a tendency to rise on account of the last added increment of current, to fall on account of the diminution of resistance due to the last but one, and to rise on account of the re-shaping of the carbons following the last but two. If the frequency of the alternating current is very low indeed, so that the current changes very slowly, all three of these tendencies will be in force at each moment, and the actual change of P.D. will be the resultant of the three. If the frequency is so high that the shapes of the carbons never change at all, but so low that the area of the volatilising surface can alter, only the first two tendencies will be operative; while, if the frequency is so high that the area of the volatilising surface remains constant, the resistance of the arc will not alter at all, the current and P.D. will increase and diminish together and proportionately, and, unless the arc contains a variable back E.M.F., $\delta V/\delta A$ will measure the true resistance of the arc.

The influence of the frequency of the alternating current on the magnitude and sign of $\delta V/\delta A$ is traced in fig. 9. PR represents the time occupied by one complete alternation, whatever that time may be. If, for instance, the frequency is 50 complete alternations per second, PR represents one-fiftieth of a second; if the frequency is 5000, PR represents one-five thousandth of a second. PSQTR represents the time change of current with any frequency. When the alternations are so slow that the arc remains normal, the change of P.D., δV, for a given small change of current, WS say, is the resultant of three such changes as BC, CD, and DE (fig. 7), and it is in the opposite direction from the change of current. The P.D. time curve is something like PXQYR (fig. 9) therefore, and $\delta V/\delta A$ is the mean of such ratios as ZX/WS, and is therefore negative.

When the frequency is raised, so that the carbons never have time to alter their shapes completely before the current changes, the third variation, DE, (fig. 7, p. 310) is smaller than with the normal arc, so that δV is greater negatively, and $\delta V/\delta A$ must, therefore, have a larger negative value than when the arc is normal, and such a curve as PX_1QY_1R would be the P.D. time curve in this case.

When the frequency was so high that the carbons never altered their shapes at all, but the volatilising surface underwent the maximum alteration, the third variation (DE fig. 7) would be absent altogether, and therefore δV would undergo the greatest change it was susceptible of in the opposite direction to the change of current, so that PX_2QY_2R is then the P.D. time curve, and $\delta V/\delta A$ has then its maximum negative value, and is the mean of such ratios as Z_2X_2/WS.

With a further increase of frequency, the area of the volatilising surface would never have time to change completely, so that δV would be the resultant of two

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such changes as BC and CF, say, (fig. 7) only; $\delta V/\delta A$ would therefore have a *smaller* negative value than with the lower frequency last mentioned, and the curve denoting the time change of P.D. might again be PX_1QY_1R, or it might be PX_3QY_3R, if the frequency were high enough. When the frequency was so great that the two P.Ds., BC and CD (fig. 7) were exactly equal, the P.D. would not alter at all when the current was changed, $\delta V/\delta A$ would be zero, and the straight line PQR would be the time change of P.D. curve. When the frequency was further increased, the change of P.D. would be the resultant of two such changes as BC, CG (fig. 7), the total change of P.D. would therefore be in the *same* direction as the change of current, the P.D.

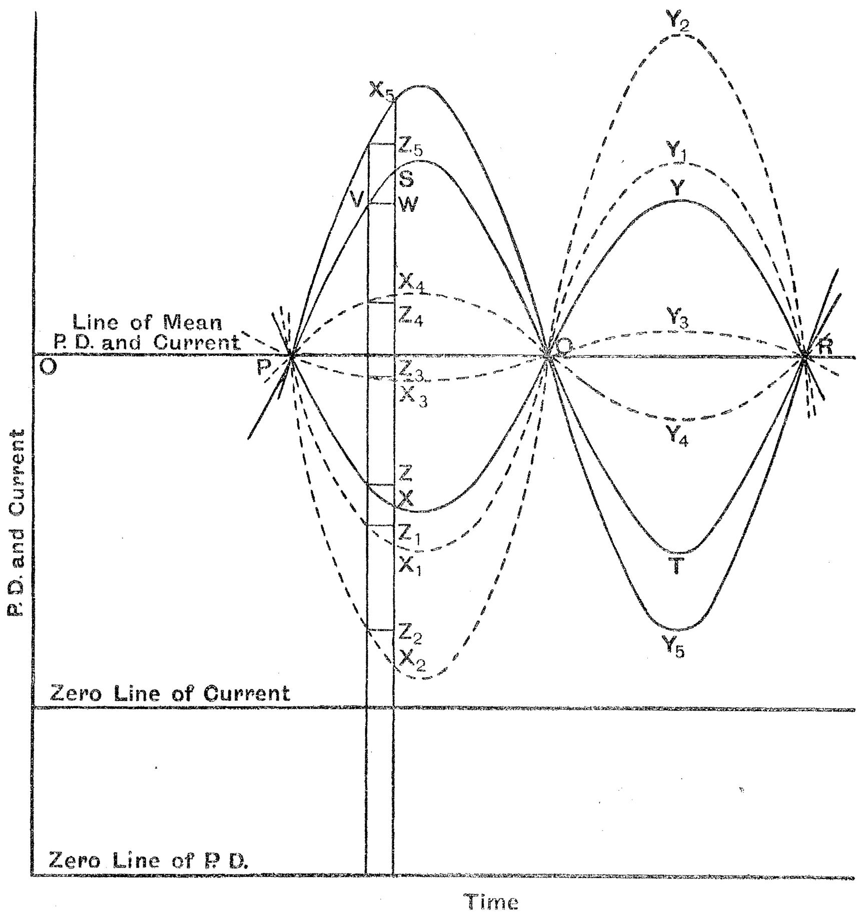

Fig 9. Suggested curves connecting current and P.D. with time for different frequencies of the same small superimposed alternating current, when the direct current and length of the arc are constant.

time curve would be like PX_4QY_4R, and $\delta V/\delta A$ would be $+ Z_4X_4/WS$. Finally, when the frequency was so great that the area of the volatilising surface never altered at all, the change of P.D. would be BC (fig. 7) alone, the P.D. time curve would be PX_5QY_5R, δV would be Z_5X_5, and $\delta V/\delta A$, or Z_5X_5/WS would measure the *true* resistance of the arc, even if there is a back E.M.F. in the arc, unless that back E.M.F. varies with the current.

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Thus, by applying the same alternating current, but with different frequencies, to a direct current arc, $\delta V/\delta A$ can be made to have any value from a fairly large negative value to the true positive value. It is easy to see, therefore, how different experimenters might get very different values and even different signs for the resistance of the arc, when they measured it by means of a superimposed alternating current; and fig. 9 shows the imperative necessity of some rigorous proof that the alternating current has not affected the resistance of the arc before any such measurements can be accepted as final. I shall presently show how such a proof can be obtained, but first it will be interesting to see how, with an arc of given length, and with a given current flowing, the value of $\delta V/\delta A$ is connected with the frequency of the alternating current, and what sort of frequency is required in order that the resistance of the arc shall not be affected by this current.

To find the Curve connecting $\delta V/\delta A$ *with the Frequency of the Superimposed Alternating Current, and to see with what Frequency* $\delta V/\delta A$ *Measures the True Resistance of the Arc.*

Take an arc of 2 millims. with a direct current of 10 amperes flowing. For the arc to remain normal when the small alternating current is superimposed on it, the frequency must be practically zero, for each alternation must take many seconds instead of only a small fraction of a second. Now the equation I have found* between V, A, and l, in the normal arc with solid Apostle carbons is

$$V = 38{\cdot}88 + 2{\cdot}07l + \frac{11{\cdot}66 + 10{\cdot}54l}{A};$$

therefore the normal $\delta V/\delta A = -\frac{11{\cdot}66 + 10{\cdot}54l}{A^2} = -0{\cdot}33$, when $l = 2$, and $A = 10$.

The first point on the curve connecting $\delta V/\delta A$ with the frequency of the alternating current has, therefore, the co-ordinates 0, and $-0{\cdot}33$ (A, fig. 10).

The value found for $\delta V/\delta A$ by Messrs. FRITH and RODGERS† with the same carbons, direct current, and P.D. was about $-0{\cdot}8$, more than double the normal value, which shows that the alternating current they superimposed was making the resistance of the arc vary to an extent that made the P.D. follow some such curve as PX_2QY_2R (fig. 9). They also found that varying the frequency from 7 to 250 complete alternations per second made no difference in the value they obtained for $\delta V/\delta A$. Therefore the curve connecting $\delta V/\delta A$ with the frequency must fall steeply from A, the point of no frequency, to B, the point for a frequency of 7, and must be practically horizontal from B to C (fig. 10). Hence Messrs. FRITH and RODGERS' observations cover the portion BC of the curve.

* 'The Electric Arc,' p. 184.

† 'Phil. Mag.,' 1896, vol. 42, Plate 5.

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The next point D, is obtained from Mr. DUDDELL'S work. In his remarkable paper* on "Rapid Variations in the Current through the Direct Current Arc," he said, "I tried to record the transient rise in P.D. for the solid arc by means of an oscillograph, the sudden increase of the current being obtained by discharging a condenser through the arc. This experiment was successful, and a transient rise in P.D. was observed, *the P.D. and current increasing together, but only for about* 1/5000 *second.*" It is clear from this that $\delta V/\delta A$ must at least *begin* to be positive with a frequency of 2,500 complete alternations per second; and D where OD = 2,500 may be taken to be the point near which $\delta V/\delta A$ changes its sign.

To the right of D the curve must continue to rise, as indicated in fig. 10, more and more slowly, as it approaches the horizontal line whose distance from the axis of frequency represents the value of $\delta V/\delta A$ which is the *true* resistance of the arc. The curve must finally become asymptotic to this line, since when once a frequency is nearly reached with which the alternating current does not practically affect the resistance of the arc, increasing the frequency will not alter the value of $\delta V/\delta A$.

My equation above shows that the resistance of the particular 2-millim. 10-ampere normal arc under discussion cannot be greater than 4·63 ohms, nor less than 0·62 ohm; for if there is no back E.M.F.,

$$r = \frac{38{\cdot}88 + 2{\cdot}07 \times 2}{10} + \frac{11{\cdot}66 + 10{\cdot}54 \times 2}{100}$$
$$= 4{\cdot}63\,;$$

and if there is the largest possible back E.M.F., viz., $38{\cdot}88 + \frac{11{\cdot}66}{A}$ volts (for it is impossible to imagine that terms involving the length of the arc can belong to a back E.M.F.), then $r = \frac{2{\cdot}07 \times 2}{10} + \frac{10{\cdot}54 \times 2}{100} = 0{\cdot}62$.

Thus the curve cannot rise higher than the horizontal line $\delta V/\delta A = 4{\cdot}63$, and it must rise at least as high as $\delta V/\delta A = 0{\cdot}62$. Consequently, as the lower curve in fig. 10 shows, the true resistance of this particular arc could not be measured with a superimposed alternating current having a frequency of less than at least 8000 complete alternations per second, even if there were a back E.M.F. as great as 40 volts. And if, as I have suggested, the back E.M.F. is zero, or at least very much smaller than 40 volts, the frequency would have to be many times as high for $\delta V/\delta A$ to be on the horizontal part of the curve, *i.e.*, for the alternating current not to alter the resistance of the arc.

The Form of the P.D. Time Curve indicates whether the Resistance of the Arc is Affected by the superimposed Alternating Current or Not.

The final test as to the frequency being high enough not to affect the resistance of the arc must, of course, be the finding, with the same root mean square value of the

* 'Journal of the Institution of Electrical Engineers,' vol. 30, p. 232.

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alternating current, of two frequencies, differing by many thousands of alternations per second, that would both give the same value of δV/δA. This would show that the horizontal part of the δV/δA frequency curve (fig. 10) had been found.

A very good *first* test, however, is furnished by the curve connecting the P.D. between the carbons with the time, for this curve is unsymmetrical with respect to the corresponding current curve, when the resistance *is* affected, for the following reasons.

I have shown that the change in the area of the volatilising surface of the crater that is due to any change of current follows *after* the change of current and requires time for its completion. If, therefore, a superimposed alternating current is affecting the resistance of a direct current arc, the P.D. required for any given current must

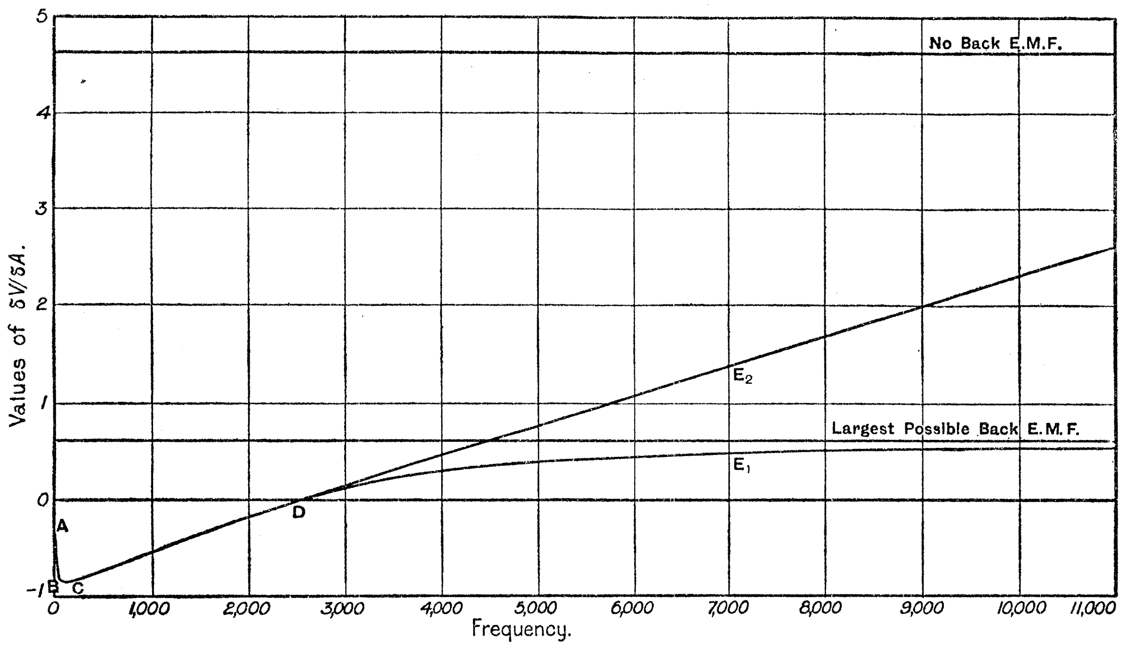

Fig. 10. Suggested curve connecting δV/δA with the frequency of a superimposed alternating current of constant root mean square value, when the direct current and the length of the arc are constant.

be higher when the current is increasing than when it is diminishing. A current of 10 amperes, for instance, would require a higher P.D. when it came after 9 and before 11 amperes than when it came after 11 and before 9 amperes, because in the first case it would be flowing through an arc of which the cross-section had been made by some current *less* than 10 amperes, and in the second by some current *greater* than 10 amperes.

I have applied this test, with very satisfactory results, to some experiments in which it is quite certain that the alternating currents must have affected the resistances of the arcs, because they had frequencies of only 47 and 115 alternations per second respectively.

The experiments formed part of a valuable series carried out in 1896 by Messrs.

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RAY and WATLINGTON, two students at the Central Technical College, in continuation of the researches of Messrs. FRITH and RODGERS. The carbons were solid, and the direct current normal arc carried a current of 10 amperes with a P.D. of 45 volts. Simultaneous values of the current and P.D. were taken with the JOUBERT point by point method. Some of these are given in Table II., in which the columns headed V_i are the P.D.'s with increasing currents, and those headed V_d with diminishing currents; while V_s and V_l belong to the smallest and largest current respectively.

It may be seen at a glance that in every single instance the P.D. for the same current is higher when the current is increasing than when it is diminishing. For instance, with the lower frequency the P.D. corresponding with a current of 11 amperes is 42·4 volts when the current is increasing, and only 41·8 volts when it is diminishing. And with the higher frequency it is 43·2 volts with increasing current, and only 42·0 with a falling current. Hence, we are supplied with a very simple test as to whether the superimposed alternating current changes the resistance of the arc or not. It is only necessary to take the wave form of P.D. and current by means of an oscillograph, and to observe whether the P.D. corresponding with each current is the same with increasing as with diminishing currents. If the two P.Ds. are different, the resistance is being altered, if they are alike, it is not.

TABLE II.—Instantaneous Values of Corresponding Currents and P.Ds. with Small Alternating Current Superimposed on Direct Current of 10 Amperes in the Arc. P.D. with Direct Current alone, 45 volts.
Solid Apostle Carbons: Positive, 11 millims.; Negative, 9 millims.

		Frequency, 47.					Frequency, 115.		
A.	V_s.	V_i.	V_l.	V_d.	A.	V_s.	V_i.	V_l.	V_d.
8·425	45·6	—	—	—	8·65	46	—	—	—
9·0	—	44·8	—	44·0	9·0	—	45·6	—	44·9
9·5	—	44·2	—	43·3	9·5	—	45·2	—	44·2
10·0	—	43·7	—	42·8	10·0	—	44·6	—	43·5
10·5	—	43·1	—	42·1	10·5	—	43·9	—	42·9
11·0	—	42·4	—	41·8	11·0	—	43·2	—	42·0
11·5	—	42·0	—	41·4	11·25	—	—	42·2	—
12·0	—	41·7	—	41·2	—	—	—	—	—
12·25	—	—	41·4	—	—	—	—	—	—

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How to Ascertain with Certainty whether there is a Constant or a Variable Back E.M.F. in the Arc or None, and how to find the True Back E.M.F. if there is One.

Returning to the equation

$$V = E + Ar,$$

we have

$$\frac{\delta V}{\delta A} = \frac{\delta E}{\delta A} + r + A\frac{\delta r}{\delta A}, \text{ when both E and } r \text{ vary,}$$

and hence

$$V - A\frac{\delta V}{\delta A} = E - A\frac{\delta E}{\delta A} - A^2\frac{\delta r}{\delta A}.$$

If the alternating current with which $\delta V/\delta A$ is measured is of such high frequency that it does not alter the resistance of the arc, and if, also, the back E.M.F. is constant, or, being variable, the alternating current is too small to affect it, then

$$V - A\,\delta V/\delta A = E.$$

To see whether the arc has any back E.M.F. at all, therefore, it is only necessary to measure $\delta V/\delta A$ with a superimposed alternating current of a frequency that has been found not to affect its resistance and to subtract $A\,\delta V/\delta A$ from V. If the result is zero, the arc has *no* back E.M.F. If it is not zero, $\delta V/\delta A$ must be measured in the same way for other arcs differing widely in current and length. If all the values of $V - A\,\delta V/\delta A$ thus obtained are equal, or nearly so, the arc has a constant back E.M.F. which is equal to this value. If $V - A\,\delta V/\delta A$ is *not* the same for all the arcs, but varies according to some definite law, then there is a *variable* back E.M.F. which may or may not be affected by the alternating current used to measure $\delta V/\delta A$.

Suppose, for instance, that two measurements of $\delta V/\delta A$ were made, using the same direct current and length of arc, but different alternating currents. If one of the alternating currents had a root mean square value equal to one per cent. of the direct current, and the other a value equal to five per cent., one would be five times as great as the other, and yet both would be small compared with the direct current. It would, of course, be possible to make the frequency of each of these currents so great that the resistances of the arcs to which they were applied were not altered by them. Yet it would not necessarily follow that when this had been done the two values of $\delta V/\delta A$ thus obtained would be equal. For the back E.M.F. might vary not with the *frequency* of the alternating current, but with its *magnitude*. If, therefore, it were found that E was variable, it would be necessary to measure $\delta V/\delta A$ with smaller and smaller alternating currents, till two were found which, while differing considerably from one another, both gave the same value of $\delta V/\delta A$. Only a value obtained in this way could be accepted as measuring the *true* resistance of the arc, and $V - A\,\delta V/\delta A$ would be the *true* back E.M.F. of the same arc.

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THE CHANGES INTRODUCED INTO THE RESISTANCE OF THE ARC BY THE USE OF CORED CARBONS.

Next let us consider the explanation of the marked effects produced by introducing a core into either or both of the carbons. These are of a two-fold character; first, those such as Professor AYRTON published at Chicago in 1893, viz.:

(1.) The P.D. between the carbons is always lower for a given current and length of arc, when either or both of the carbons are cored, than when both are solid.

(2.) With a constant length of arc and increasing current, the P.D., which diminishes continuously when both carbons are solid, either diminishes less when the positive is cored, or after diminishing to a minimum remains constant over a wide range of current, or even increases again.*

(3.) It requires a larger current with the same length of arc to make the arc hiss when the positive carbon is cored than when both are solid.

Secondly, there are the facts connected with the influence of cores on the small *change* of P.D. accompanying a small *change* of current, to which attention was first drawn by Messrs. FRITH and RODGERS in 1896.† These facts, which were physically correct, although, as I have already shown, they were wrongly interpreted at the time, are embodied in the following wider generalisations which I have deduced from the results of my experiments, and from theoretical considerations.

(1) When, on a direct current arc, an alternating current is superimposed which is small, but yet large enough for the resistance of the arc to be altered by it, the average value of $\delta V/\delta A$ is always more positive‡ when either carbon is cored than when both are solid, and most positive of all when both are cored, all other things being equal.

(2) The frequency of the alternating current that makes $\delta V/\delta A$ begin to have a positive value is lower when either carbon is cored than when both are solid, and lowest when both are cored.

(3) The value of $\delta V/\delta A$, with a given root mean square value of the superimposed alternating current, depends not only on the nature of the carbons and on the frequency of that current, but also on the magnitude of the direct current, and on the length of the arc.

* NEBEL observed the fact that the P.D. fell to a minimum and then rose again, in 1886, but as he used *cored* positive carbons *only*, he did not discover that this form of curve was peculiar to those carbons.

† "The Resistance of the Electric Arc," 'Phil. Mag.,' 1896, vol. 42, p. 407.

‡ I call $\delta V/\delta A$ *more positive* in one case than in another when it has either a larger positive value, or a smaller negative value in the first case than in the second.

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There are two ways in which the P.D. between the carbons, for a given current and length of arc, may be lowered by the core; (1) by an increase in the cross-section of the vapour film, or the mist, or both; (2) by a lowering of their specific resistances. To see whether I could observe any change in the cross-sections, I have traced a series of enlarged images of the arc with four sets of Apostle carbons, using (1) + Solid − Solid, (2) + Solid − Cored, (3) + Cored − Solid, (4) + Cored − Cored carbons.

The positive carbon was 11 millims. and the negative 9 millims. in diameter, and the arc 2 millims. in length in each case, while the currents were 4, 6, 8, 10, 12, 14 amperes. The diagrams were traced not only when the arc was normal in each case, but also immediately after each change of current, so that the effect on the cross-section of the arc of both an instantaneous and a normal change of current might be seen. Fig. 5 (p. 307) showed the first set of diagrams of the normal arc; the others are too numerous to publish, but the mean cross-sections of the purple part—the mist—in each, measured as in fig. 5, may be found in Table III., those marked "non-normal" belonging to the arc *immediately* after the change of current, and those marked "normal" to the arc after all the conditions had become steady again.

TABLE III.—Mean Cross-Sections of Mist between + Solid − Solid, + Solid − Cored, + Cored − Solid, and + Cored − Cored Apostle Carbons, 11 millims. and 9 millims.

Length of Arc, 2 millims.

Current in amperes.	Normal.				Non-normal.			
	S.S.	S.C.	C.S.	C.C.	S.S.	S.C.	C.S.	C.C.
4	4·8	6·95	4·0	3·3	—	—	—	—
6	9·8	8·3	6·05	5·6	9·5	8·4	6·25	3·5
8	16·2	14·2	11·0	8·9	17·6	11·1	12·0	5·8
10	23·4	20·75	13·55	11·9	21·5	19·0	18·7	11·1
12	34·9	27·6	17·7	16·55	34·1	26·9	20·1	16·7
14	41·2	35·0	24·5	20·0	—	39·4	—	18·7

With a single exception, every number in each set is smaller than the corresponding number in the preceding column. Hence, with both normal and non-normal arcs the mean cross-section of the mist, for a given current, is largest when both carbons are solid, smallest when both are cored, and is more diminished by coring the positive than by coring the negative. Fig. 11, besides showing well this marked difference in the influence of the cores, makes it apparent that the difference increases, in every case, with the current, for such currents as were there used.

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We cannot measure the cross-section of the vapour film directly, but, for a constant length of arc, it must be roughly proportional to the cross-section of the mist where it touches the crater. These cross-sections, which are given in Table IV., do not, naturally, vary nearly so regularly as the mean cross-sections, but still we can judge pretty well what are the effects of the various cores. Coring the positive carbon, for instance, distinctly diminishes the cross-section of the vapour film; for every

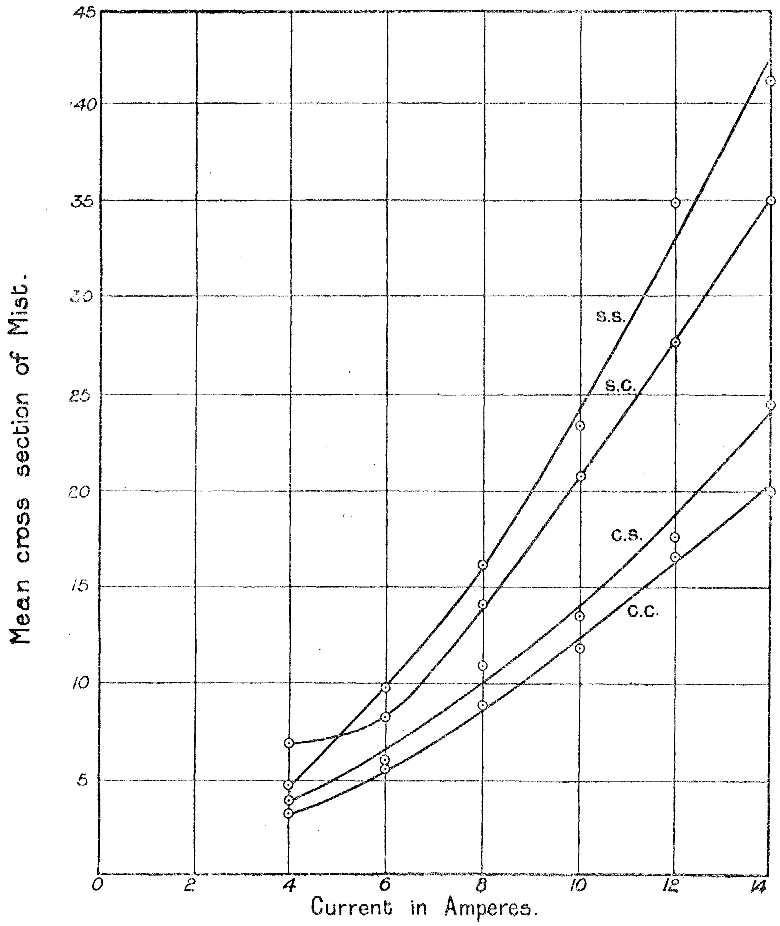

Fig 11. Curve connecting the mean cross-section of the arc mist with the current for + solid, – solid, + solid – cored, + cored – solid, and + cored – cored carbons respectively. Apostle carbons, 11 millims. and 9 millims. Length of arc, 2 millims.

number in column (7) is less than the corresponding one in column (5), and all but one in column (3) are less than those in column (1). Coring the negative carbon, on the other hand, only seems to affect the cross-section that the vapour film assumes *immediately* after a change of current, for while in the *non-normal* section each number in (6) is less than in (5), and in (8) less than in (7), in the *normal* section the numbers in (2) are sometimes less and sometimes greater than those in (1), and those in (4) are nearly all greater than those in (3).

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TABLE IV.—Cross-section of Mist where it touches Crater, with + Solid — Solid, + Solid — Cored, + Cored — Solid, + Cored — Cored Carbons.

APOSTLE Carbons, 11 millims. and 9 millims.

LENGTH of Arc, 2 millims.

Current in Amperes.	Normal.				Non-normal.			
	(1) S.S.	(2) S.C.	(3) C.S.	(4) C.C.	(5) S.S.	(6) S.C.	(7) C.S.	(8) C.C.
4	2·9	7·8	3·2	2·9	—	—	—	—
6	6·8	9·0	5·3	5·8	10·9	6·25	6·25	3·6
8	16·0	13·0	10·9	14·4	16·0	9·0	10·9	6·25
10	23·0	26·0	12·25	21·2	19·4	16·8	15·2	15·2
12	32·5	25·0	17·6	22·1	31·4	23·0	17·6	21·2
14	39·1	36·0	23·0	24·0	—	33·6	—	19·4

Thus, taking Tables III. and IV. together, we find that a core in the positive carbon keeps both the mist and the vapour film from being as large as they would be with a solid positive, both immediately after a change of current and when the arc is normal again. Coring the negative, on the other hand, while it has the same effect on the cross-section of the *mist* as coring the positive, only diminishes the cross-section of the *vapour film* immediately after a change of current. If, therefore, coring either carbon produced nothing but an alteration in the cross-section of the arc, the resistance of the arc, and, consequently, the P.D. between the carbons would be *increased* by the coring. It follows, therefore, that the diminution of the P.D. between the carbons actually observed with cored carbons must be caused by a lowering of the specific resistance of the vapour film or of the mist, or of both; and this lowering must be so great that it must more than compensate for the diminution in their cross-sections.

It is easy to see how the vapour and mist from a core in the *positive* carbon must alter the specific resistance of the arc, but, since the *negative* carbon does not volatilise, there seems to be no reason why coring *it* should have the same effect. The core, however, consists of a mixture of carbon and metallic salts; and metallic salts have a lower temperature of volatilisation than carbon, so that these salts may easily be volatilised by the *mist* touching them, and, mingling with it, lower its specific resistance.

Now take the fact that, with a constant length of arc, on increasing the current the P.D. always diminishes less if the positive carbon is cored than if it is solid, and that the reduction of diminution is sometimes so great that the P.D. remains constant for a large increase of current, and sometimes even increases somewhat, instead of steadily diminishing, as it does when both carbons are solid.

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Every increase of current, whether the carbons are solid or cored, entails an enlargement of the cross-section of the arc, and a consequent tendency of the P.D. to diminish. While the current is so small, with cored carbons, that the volatilised surface does not completely cover the core, the increase of cross-section is unaccompanied by any change in the specific resistance of the arc. When the current is so large, however, that the solid carbon round the core begins to volatilise, each increase

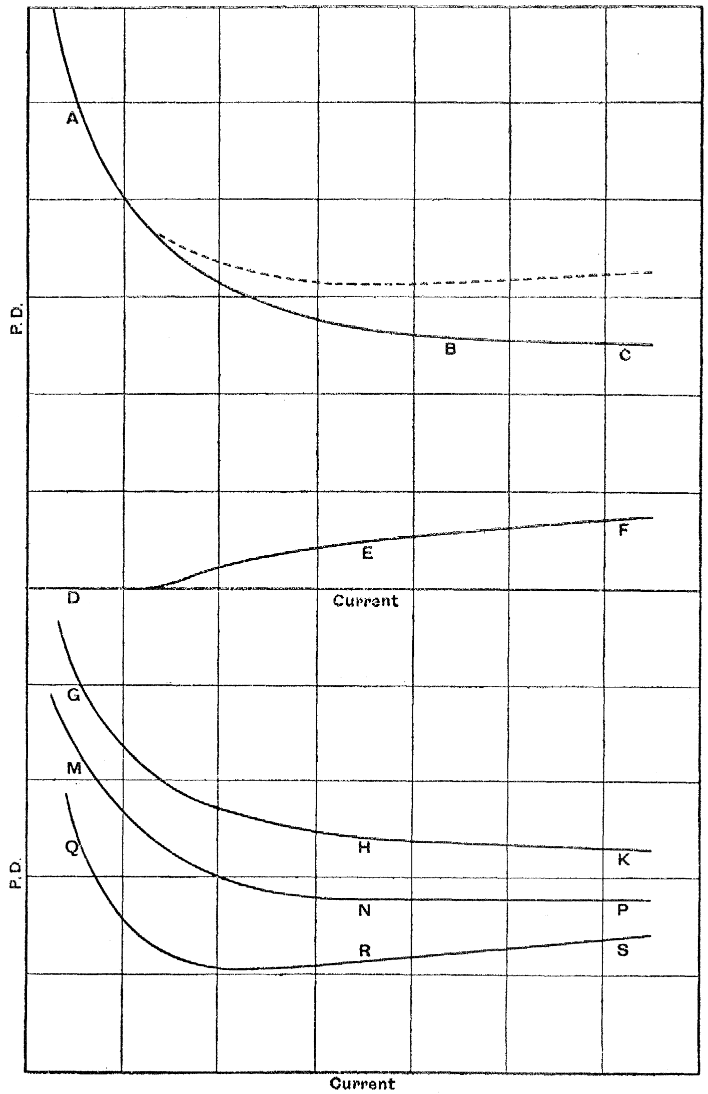

Fig. 12. Curves exemplifying the changes introduced into the curves connecting P.D. with current for a constant length of arc by coring the positive carbon.

of current is accompanied by two tendencies in the P.D., the one to *fall*, on account of the larger cross-section, the other to *rise*, because of the higher specific resistance of the vapour and mist coming from the solid portion of the carbon. The curve connecting the P.D. with the current must, therefore, be compounded of two. One, such as A B C (fig. 12), which would connect the P.D. with the current if the positive

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carbon were composed entirely of core, and the other, D E F, connecting the rise of P.D., due to the increase of the specific resistance, with the current. The curve connecting the true P.D. with the current is found by adding each ordinate of D E F to the corresponding ordinate of A B C, as indicated in the dotted line. Whether this resulting curve has the form G H K, or M N P, or Q R S (fig. 12) depends, evidently, upon the relation between the increase of the cross-section and the rise of specific resistance, *i.e.*, on the relative structures and cross-sections of the core and the outer carbon.

The fact, already obtained from Table IV., that, for the same current and length of arc, the vapour film, and, consequently, the crater, is smaller with a cored than with a solid positive carbon, explains why the arc can carry such a much larger current without hissing when the positive carbon is cored than when it is solid. For I have shown* that hissing is the result of that direct contact of the crater with the air which follows when the crater grows too large to cover the end only of the positive carbon and so extends along its sides, and this must happen with a smaller current the larger the crater is with a given current, *i.e.*, it must happen with a smaller current when the positive carbon is solid than when it is cored.

How the Change in the Cross-Sections of the Mist and the Vapour Film, due to a Change of Current, is Affected by Coring Either or Both Carbons.

Consider next the facts concerning the influence of the cores on the values of $\delta V/\delta A$, when a small alternating current is superimposed on a direct current *normal* arc, that the resistance of the arc is affected by this superposition. Here we have to deal, not with the whole P.D. between the carbons, but with the *change* in that P.D. that accompanies a given small *change* of current, and I shall show that the effect of the core on this change must always be to *add* a positive increment to $\delta V/\delta A$, the amount of which depends on the value of the direct current, the length of the arc, and the frequency of the alternating current.

The influence of the core on the value $\delta V/\delta A$ is two-fold; it alters the amount by which the cross-sections of the vapour film and the mist change, with a given change of current; and it makes their specific resistances vary with the current. We will take each separately, the change of cross-section first. I shall call the part of $\delta V/\delta A$ that depends on the change of cross-section $\delta V_c/\delta A$, and the part that depends on the variation in the specific resistance $\delta V_s/\delta A$, so that $\delta V_c/\delta A + \delta V_s/\delta A = \delta V/\delta A$.

I have already pointed out (p. 306) that if, when the current is increased, the ratios of the new cross-sections of the mist and the vapour film to the old are greater than

* "The Hissing of the Electric Arc," 'Journal of the Proceedings of the Institution of Electrical Engineers,' 1899, vol. 28, p. 400.

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the ratio of the new current to the old, then the resistance of the arc must have been diminished more than the current was increased, and that hence the P.D. must have diminished as the current increased, and $\delta V/\delta A$ must be negative (provided always that the specific resistance of the arc had not been altered). Similarly, when the ratio of the cross-sections are smaller than that of the current $\delta V/\delta A$ must be positive.

In order to see the effect of the cores on these ratios, in the experiments of which the results are given in Tables III. and IV., Tables V. and VI. have been drawn up, in which the cross-section ratios are found by dividing the cross-section for each current by the cross-section for the next smaller current; and the current ratios by dividing each current by the next smaller current. For the non-normal ratios the larger cross-sections were taken from the *non-normal* sets in Tables III. and IV. and the smaller from the *normal*, because it is the effect of the core when the alternating current is superimposed on a *normal* arc that we are considering, and because also the non-normal numbers in these Tables were found by suddenly *increasing* the current when the arc was normal. For the normal ratios *both* currents were taken from the normal sets.

For instance, the normal cross-section for a current of 8 amperes with the + solid-— cored carbons in Table III. (p. 321) is 14·2, and the non-normal cross-section for 10 amperes is 19·0, while the normal cross-section for the same current is 20·75. Thus, when the current is increased from 8 to 10 amperes, the current ratio is $\frac{10}{8} = 1{\cdot}25$, the non-normal cross-section ratio with these carbons is $\frac{19}{14{\cdot}2} = 1{\cdot}34$, and the normal is $\frac{20{\cdot}75}{14{\cdot}2} = 1{\cdot}46$. In this case, therefore, $\delta V_c/\delta A$ was negative, both when the change of current was non-normal and when it was normal, for both the cross-section ratios, 1·34 and 1·46, are greater than 1·25, the current ratio. Of course to imitate the effect of an alternating current completely it would be necessary to suddenly diminish the current as well as suddenly increasing it, but as this would only alter the signs of both δV and δA, without materially changing their relative values, it is not necessary for our purpose.

The non-normal ratios show the effect of the core on the change in the cross-section, *i.e.*, in the resistance of the arc, and therefore on δV, when the frequency of the alternations is so great that the carbons do not change their shapes; and the normal, when it is so small that the changes are slow enough for the arc to rema n practically normal throughout.

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TABLE V.—Ratios of Mean Cross-Sections of Mist taken from Table III.

LENGTH of Arc, 2 millims.

Change of Current.	Current Ratios.	Ratios of Mean Cross-Sections.							
		Normal.				Non-normal.			
		S.S.	S.C.	C.S.	C.C.	S.S.	S.C.	C.S.	C.C.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
4 to 6	1·5	2·04	1·2	1·51	1·7	1·98	1·21	1·56	1·06
6 ,, 8	1·33	1·65	1·71	1·82	1·6	1·8	1·33	1·98	1·04
8 ,, 10	1·25	1·44	1·46	1·23	1·33	1·33	1·34	1·70	1·25
10 ,, 12	1·20	1·49	1·33	1·31	1·39	1·46	1·30	1·48	1·40
12 ,, 14	1·17	1·18	1·25	1·38	1·21	—	1·43	—	1·13

The most important point to observe in these tables is whether $\delta V_c/\delta A$ is negative or positive with each set of carbons, *i.e.*, whether the cross-section ratios are greater or less than the current ratios. Take first the non-normal ratios. When the positive carbon alone is cored, the *sign* of $\delta V_c/\delta A$ is decidedly negative, just as it is when both carbons are solid, for all the cross-section ratios in column (9) of both Tables V. and VI. are greater than the corresponding current ratios in column (2). Moreover, with this particular length of arc, and these currents, the non-normal *value* of $\delta V_c/\delta A$ does not appear to be altered by coring the *positive* carbon alone, for the non-normal cross-section ratios in column (9) of each table are in some cases greater and in others less than those in column (7). When the *negative* carbon alone is cored, the non-normal value of $\delta V_c/\delta A$ appears to be negative, but approaching the zero point; for in Table V., one cross-section ratio in column (8) is less than the corresponding current ratio, one is equal, and three are greater; while in Table VI., three are less and two are greater. When *both* carbons are cored, the non-normal value of $\delta V_c/\delta A$ is positive; for three out of the five of the numbers in column (10) of Table V., and the whole of those in the same column of Table VI. are greater than the corresponding numbers in column (2).

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TABLE VI.—Ratios of Cross-Sections of Mist where it Touches the Crater, taken from Table IV.

LENGTH of Arc, 2 millims.

Change of Current.	Current Ratios.	Ratios of Cross-Sections at Crater.							
		Normal.				Non-normal.			
		S.S.	S.C.	C.S.	C.C.	S.S.	S.C.	C.S.	C.C.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
4 to 6	1·50	2·34	1·07	1·66	2·00	3·76	0·80	1·95	1·24
6 „ 8	1·33	2·35	1·44	2·05	2·50	2·35	1·00	2·06	1·08
8 „ 10	1·25	1·44	2·00	1·12	1·47	1·21	1·29	1·40	1·05
10 „ 12	1·20	1·41	0·96	1·44	1·04	1·37	0·88	1·44	1·00
12 „ 14	1·17	1·20	1·44	1·31	1·09	—	1·34	—	0·88

Turning next to the normal ratios, we find that when the positive carbon alone is cored, $\delta V_c/\delta A$ has still much the same negative value as when both carbons are solid, since the numbers in column (5) differ on the whole very little from those in column (3). When, on the other hand, it is the negative carbon alone that is cored, there is a change, for instead of being a little below zero, $\delta V_c/\delta A$ is decidedly negative, since in Table V. all but one of the numbers in column (4), and in Table VI. all but two are greater than the corresponding numbers in column (2). When *both* carbons are cored, there is an even greater difference between the normal and non-normal values of $\delta V_c/\delta A$. For in Table V. all the numbers in column (6), and in Table VI. all but two are greater than the corresponding current ratios, showing that $\delta V_c/\delta A$ is *negative* for normal changes of current though it is positive for non-normal changes with these carbons, currents, and length of arc.

To sum up the change in the value of $\delta V_c/\delta A$ produced by coring one or both of the carbons, we find that while coring the positive carbon alone makes very little difference in either the normal or non-normal change of cross-section that accompanies a given change of current, coring the negative carbon *diminishes* the change of cross-section, both for normal and non-normal changes of current, but more for the second than for the first, and more when both carbons are cored than when the negative alone is cored. Thus, coring the negative carbon both *diminishes* and *retards* the change in the cross-sections of the arc that accompany a change of current. This retardation of the change of cross-section is quite sufficient to account for the fact already mentioned on p. 311, viz., that if I quickly altered the resistance in the circuit outside the arc, when both carbons were *cored*, I could sometimes see

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the first quick swing of the voltmeter needle in the same direction as that of the ammeter, but never when both were *solid*. For as the resistance did not alter *directly* after the current, with the cored carbons, the new current would be flowing through the old resistance for an appreciable time after the change, and so the accompanying change of P.D. in the same direction as the change of current would be able to influence the voltmeter needle.

The Change in the Specific Resistance of the Arc produced by a Change of Current when Either or Both Carbons are Cored.

We have next to consider $\delta V_s/\delta A$, the part of $\delta V/\delta A$ that depends on the changes in the specific resistances of the mist and the vapour that occur with each change of current, when either or both carbons are cored.

As it is the positive carbon only that *volatilises,* while the negative simply burns, coring the negative carbon alone must have a very different effect on the specific resistance of the arc from coring the positive alone. For when the negative carbon alone is cored, the whole of the vapour and almost the whole of the mist must issue from the *uncored* carbon, the core in the negative carbon only contributing a little metallic vapour to the mist in contact with it; when, on the other hand, it is the positive alone that is cored, the whole comes from the *cored* carbon. Thus, while, with the cored *negative*, the vapour is always *solid-carbon* vapour, and the mist is practically *solid-carbon* mist, with the cored *positive* the vapour and mist are both *core* vapour and mist alone, until the current is large enough for the volatilising surface to cover the whole core, and they only begin to have an admixture of solid-carbon vapour and mist when the current is larger than this. When, therefore, the *negative* carbon alone is cored, the specific resistance of the vapour is constant, and that of the mist increases with each small increase of current, but more and more slowly, with the *same* addition of current, the larger the original current before the addition is made. The curve connecting $\delta V_s/\delta A$ with the normal current in this case must, therefore, be of the form A B C (fig. 13), for the *change* of specific resistance must be greatest when the current is just large enough for the mist to cover the whole core, and must steadily diminish as the direct current increases after that, till it becomes practically zero with very large currents, so that the curve becomes asymptotic to the axis of current.

When the positive carbon alone is cored, the curve is quite different. If the arc always remained perfectly central, it would be of the form D E F G (fig. 13). The specific resistances of the vapour and mist would remain constant till the volatilising surface was large enough to cover the core, so that until then $\delta V_s/\delta A$ would be zero, and D E would be the first part of the curve. The first increment of current that was added after this would increase the specific resistances by the largest possible

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amount, because this would be the point at which the specific resistances of the existing vapour and mist and of those added would be most different. Therefore the curve would rise suddenly at E. After this, each addition to the normal current would make the change of specific resistance due to the same added small non-normal increment of current smaller and smaller, so that the curve would fall towards the axis of current as shown in F G (fig. 13). Finally, there would already be so much solid-carbon vapour and mist in the arc that the addition of a little more would make practically no change, so that this curve also is asymptotic to the axis of current. The fact that the arc is never really quite central, and that the volatilising surface must therefore cover a little solid carbon long before it is larger than the core, must introduce some modifications into the first part of the curve, shortening D E, and making E F rise less abruptly, something like D E′ F′ G, but these modifications are unimportant.

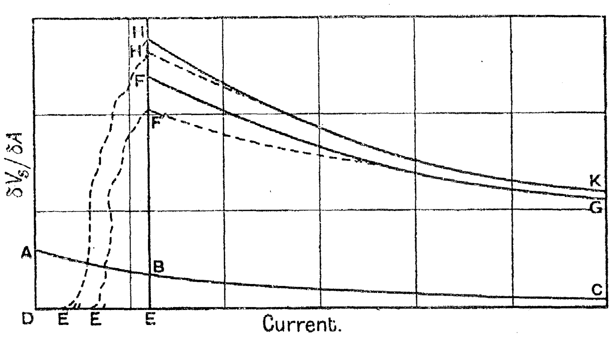

Fig. 13. Suggested curves connecting the part of $\delta V/\delta A$ that depends on the change in the specific resistance of the arc with the direct current for a constant length of arc.

When both carbons are cored, the curve must be like D E H K, or rather D E″ H′ K, because the effect of the metallic vapour from the negative core will be added to that of the positive core, and the change of specific resistance, when solid-carbon mist begins to be added, will, therefore, be greater.

How the Whole Value of $\delta V/\delta A$ *is Affected by Coring Either or Both Carbons.*

By combining the two changes in the resistance of the arc introduced by the core, viz., that due to the changes in the cross-sections of the arc, and that produced by the alterations in its specific resistance, we can see how the complete value of $\delta V/\delta A$ is affected by the core.

From what has been said on p. 329 it is clear that, if the cross-section ratios in Tables V. and VI. can be considered typical, $\delta V_c/\delta A$ never has a greater negative value when the positive carbon alone is cored than when both are solid; never a greater negative value when the negative alone than when the positive alone is

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cored, and never a greater negative value when both are cored than when the negative alone is cored, the conditions as to current length of arc, &c., being the same in all cases. But $\delta V_s/\delta A$ is zero when both carbons are solid, is greatest when both are cored, and has always some positive value, however small, when either carbon alone is cored. Consequently, when the superimposed alternating current alters the resistance of the arc, if all other things are equal, the sum of these two, *i.e.*, $\delta V/\delta A$, is more positive when either carbon is cored than when both are solid, and most positive when both are cored.

The general effect on $\delta V/\delta A$ of coring either or both carbons is given in the preceding paragraph, but with a given root mean square value of the alternating current $\delta V/\delta A$ depends not only on the nature of the carbons, but also on the frequency of the alternating current, the magnitude of the direct current, and the length of the arc. To complete our knowledge of the influence of cores on the value of $\delta V/\delta A$ therefore, we must examine the effect they produce on the curves connecting each of these variables with $\delta V/\delta A$ when the others are constant. Take first the curves connecting $\delta V/\delta A$ with the frequency of the alternating current.

The Change Produced in the Curve Connecting $\delta V/\delta A$ *with the Frequency of the Alternating Current, by Coring Either or Both Carbons.*

A B C (fig. 14), which is copied from fig. 10, is the curve, connecting $\delta V/\delta A$ with the current-frequency for solid carbons. Since for moderate frequencies $\delta V/\delta A$ is always most positive when both carbons are cored, and more positive when one is cored than when both are solid, the curve when both carbons are cored must resemble D E F, and the curves for one carbon cored and the other solid must lie between A B C, and D E F, but we have no means of knowing which of the two will start the higher. It follows, therefore, that the frequency with which $\delta V/\delta A$ becomes positive, if it is not already positive, for normal changes of current (*i.e.*, for frequency 0), must be lower when one carbon is cored than when both are solid and lowest when both are cored. Thus, with the same direct current and length of arc, $\delta V/\delta A$ may be positive for all four sets of carbons, as at the points C, P, K, and F, or positive for some and negative for others as at B N H and E, or negative for all. Moreover, since the *true* resistance of the arc is greatest when both carbons are solid and least when both are cored, and smaller when the positive alone than when the negative alone is cored, the *horizontal* part of the curve, which shows the true resistance of the arc, must be highest when both carbons are solid, next highest for + solid − cored carbons, lower for + cored − solid carbons, and lowest of all when both are cored. Hence, since the curve for two solid carbons starts lowest, it must cut all the others at some fairly high

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frequencies, and that for two cored carbons, which starts highest, must also cut the other two, so that the curves will be like I (fig. 14), if the curve starts lower when

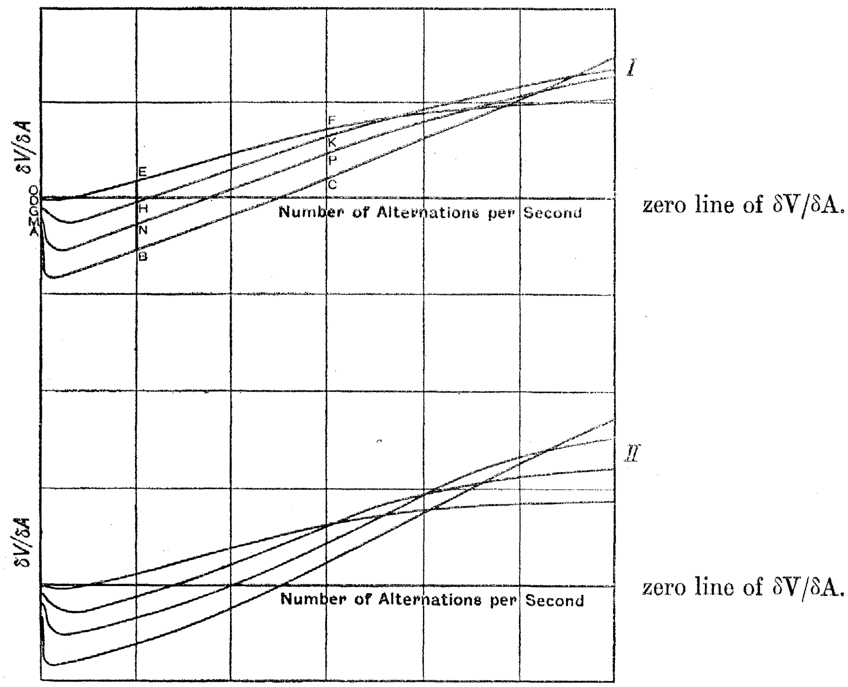

Fig. 14. Suggested curves connecting $\delta V/\delta A$ with the frequency of the superimposed alternating current for a constant direct current and length of arc.

the positive alone is cored, than when the negative alone is cored; and like II (fig. 14) if it starts higher.

The Effect Produced by Coring Either or Both Carbons on the Curve Connecting the Non-Normal Value of $\delta V/\delta A$ *with* A, *when the Length of the Arc is Constant.*

Take next the curves connecting $\delta V/\delta A$ with the normal direct current, when the length of the arc and the frequency of the alternating current are constant.

In Tables V. and VI., pp. 327, 328, the cross-section ratios for solid carbons differ less, on the whole, from the corresponding current ratios the larger the current on which the increase of 2 amperes has been superimposed. This shows that with solid carbons, when the length of the arc is constant, $\delta V/\delta A$ diminishes as the current increases. Consequently the curve connecting $\delta V/\delta A$ with A for solid carbons is of the form A B C (fig. 15). With cored carbons the curves depend not only on $\delta V_c/A$, which is obtained from Tables V. and VI., but also on $\delta V_s/\delta A$, the curves connecting which with A are given in fig. 13. The curves connecting $\delta V_c/\delta A$ with A cannot be obtained straight from Tables V. and VI., because the values are too irregular, but we can

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deduce them from what we already know. For instance, when the positive carbon alone is cored, it must have the same form A B C, as when both are solid, since the change of cross-section due to a given change of current is not materially altered by coring the positive carbon alone. Coring the negative carbon alone *diminishes* the negative value of $\delta V_c/\delta A$, and must diminish it most when the current is least, for it is then that the metallic vapour from the core will be expended on the smallest

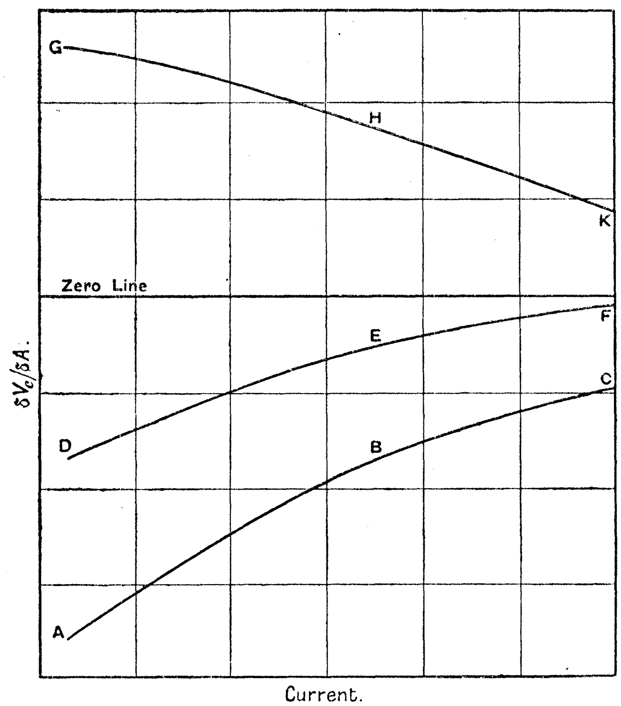

Fig. 15. Suggested curve connecting the part of $\delta V/\delta A$ that depends on changes in the cross-section of the arc with the direct current for a constant length of arc.

quantity of hard carbon mist, and will consequently have most effect. Hence the curve for a cored negative and solid positive carbon must resemble D E F (fig. 15), and the current for which $\delta V_c/\delta A$ becomes positive, if any, will depend upon the length of the arc and the frequency for which the curve is drawn. Finally, with both carbons cored, $\delta V_c/\delta A$ is even more positive than when the negative only is cored, so that the curve must resemble G H K (fig. 15), since the same reasoning as before shows that the cores have least effect when the current is largest.

To find the full curves connecting $\delta V/\delta A$ with A, for each pair of carbons, we have only to add each ordinate of each curve in fig. 13 to the corresponding ordinate of the curve for the same carbons in fig. 15. Curves resembling those that would be thus obtained for one length of arc and frequency of alternating current are given in fig. 16. The exact distance of each above or below the zero line, and the exact points where it cuts that line must, of course, depend upon the length of arc and frequency

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of alternating current for which the curves are drawn, but their relative shapes and positions must be similar to those in fig. 16 whatever the length of the arc and the frequency.

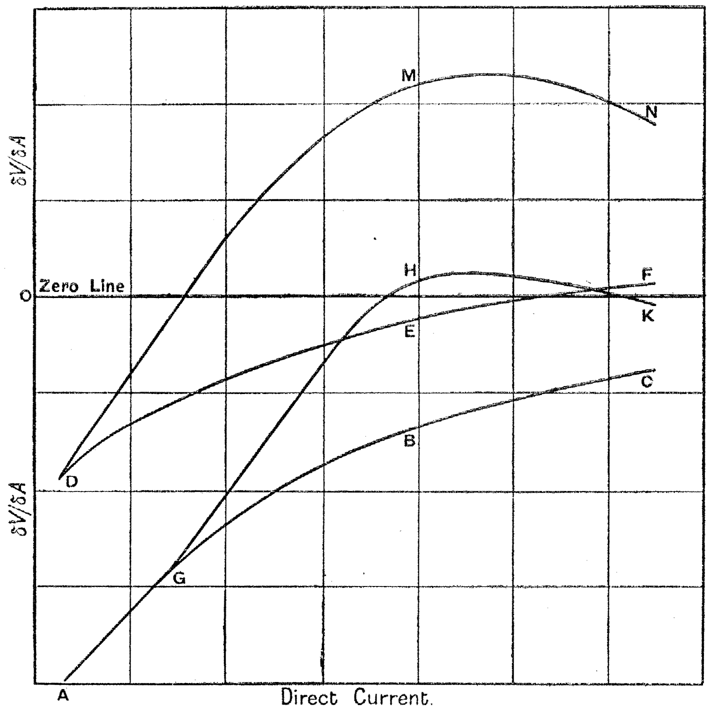

Fig. 16. Suggested curves connecting $\delta V/\delta A$ with the direct current for a constant length of arc.

The Effect Produced by Coring Either or Both Carbons on the Curve connecting the Non-Normal Value of $\delta V/\delta A$ *with the Length of the Arc when* A *is Constant.*

Finally, we come to the curve connecting $\delta V/\delta A$ with l, the length of the arc, when the frequency of the alternating current and the value of the direct current are both constant.

We must refer first of all to the connection between $\delta V/\delta A$ and l when both carbons are solid, in order to see how this connection is varied by the cores. P Q (fig. 17) is the rise of P.D. that would accompany the increase of current δA, with an arc of l millims., if the resistance of the arc did not alter with the current. Q R is the fall of P.D. due to the enlargement of the vapour film and the mist (the frequency of the superimposed alternating current is taken too great for the carbons to be able to change their forms). When the current increases from A to A + δA, therefore, the P.D. actually falls from P to R. Now the rise P Q depends only on the amount by which the current is increased, and the resistance through which that increased current has to flow, *i.e.*, on δA, A, and l; or, since A and δA are supposed to be the same for each length of arc, P Q depends simply on l, and increases when l increases.

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The *fall* of P.D.—Q R—is more complex. It depends principally on how much of the extra carbon volatilised by the larger current remains between the carbons, and how much escapes along them. When the carbons have short thick ends more will remain than when they have long pointed ones, and as the ends of the carbons are thicker, with the same current, the longer the arc, a small increase of current will diminish the resistance of the arc more, the longer the arc. But the blunting of the carbons, which is a rapid affair when the arc is short, takes place more and more slowly as it is lengthened, till at last the addition of a millimetre or so to the length of the arc makes practically no difference in the shapes of the carbons. Hence the diminution of resistance due to the addition of δA to the current increases rapidly at first, when the arc is short, and more and more slowly as the arc lengthens, till finally it becomes practically constant; and hence also, Q R—the fall of P.D. accompanying this diminution—increases more and more slowly as the arc is lengthened. Thus, while the rise of P.D.—P Q—increases at a *constant* rate as the arc is lengthened, the fall, Q R, increases at a *diminishing* rate. While the arc is so short, therefore, that Q R increases more rapidly than P Q when l is increased, the whole fall of P.D.—P S—will *increase,* with the length of the arc, or, since P S is $-\delta V$ and δA is the same for all the lengths of arc, $-\delta V/\delta A$ *increases* as the arc is lengthened. When the arc is so long that P Q increases faster than Q R, P S, and, therefore, $-\delta V/\delta A$ will *diminish* as the arc is lengthened. Between the two stages there must be a length of arc for which $-\delta V/\delta A$ is a maximum. The curve connecting $-\delta V/\delta A$ with l for a constant current, with solid carbons, must, therefore, be of the form A B C (fig. 18), and *there seems to be no reason why, with very long arcs,* $\delta V/\delta A$ *should not actually become positive, with superimposed alternating currents of comparatively low frequency, even with solid carbons.*

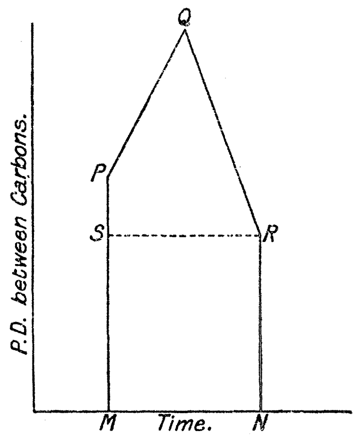

Fig. 17.

The curves connecting $\delta V/\delta A$ with l, when cored carbons are used, must resemble the curve for solid carbons, A B C (fig. 18), but must be higher up the figure (D E F, G H K) when one carbon is cored, and still higher up (M N P) when both are cored. Also, since a change in the specific resistance of the arc must have more effect on the value of $\delta V/\delta A$ the longer the arc, the distance between the curves for cored carbons and the curve for solid carbons must increase as the arc is lengthened, as it is made to do in fig. 18.

Curves very similar to those in figs. 16 and 18 were obtained by Messrs. FRITH and RODGERS, in 1896, by actual measurements of $\delta V/\delta A$. Other measurements that they carried out at the same time coincide with some of the other deductions I have made concerning the influence of cores on the value of $\delta V/\delta A$. Hence, experience, as

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far as it goes, confirms the conclusions to which I have been led by a theoretical consideration of the conditions. Since, therefore, all the principal phenomena of the arc but one,* with cored and with solid carbons alike, may be readily accounted for

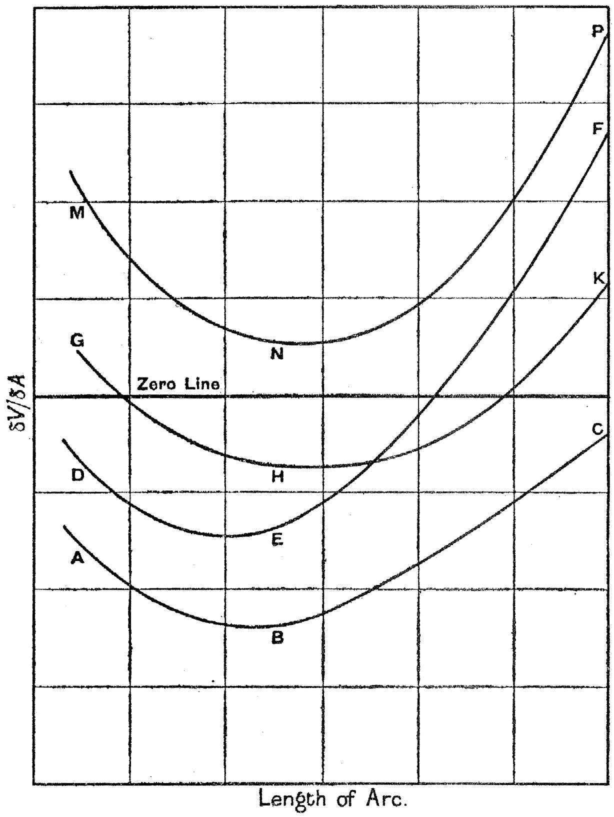

Fig. 18. Suggested curves connecting δV/δA with the length of the arc for a constant direct current

without recourse to any such unusual attributes as a negative resistance, or even a large back E.M.F., it seems superfluous to imagine their existence without stronger proof of it than has yet been obtained.

* The one exception is the fall of potential of some 8 to 11 volts at the junction of the negative carbon with the arc. This may be a true back E.M.F.

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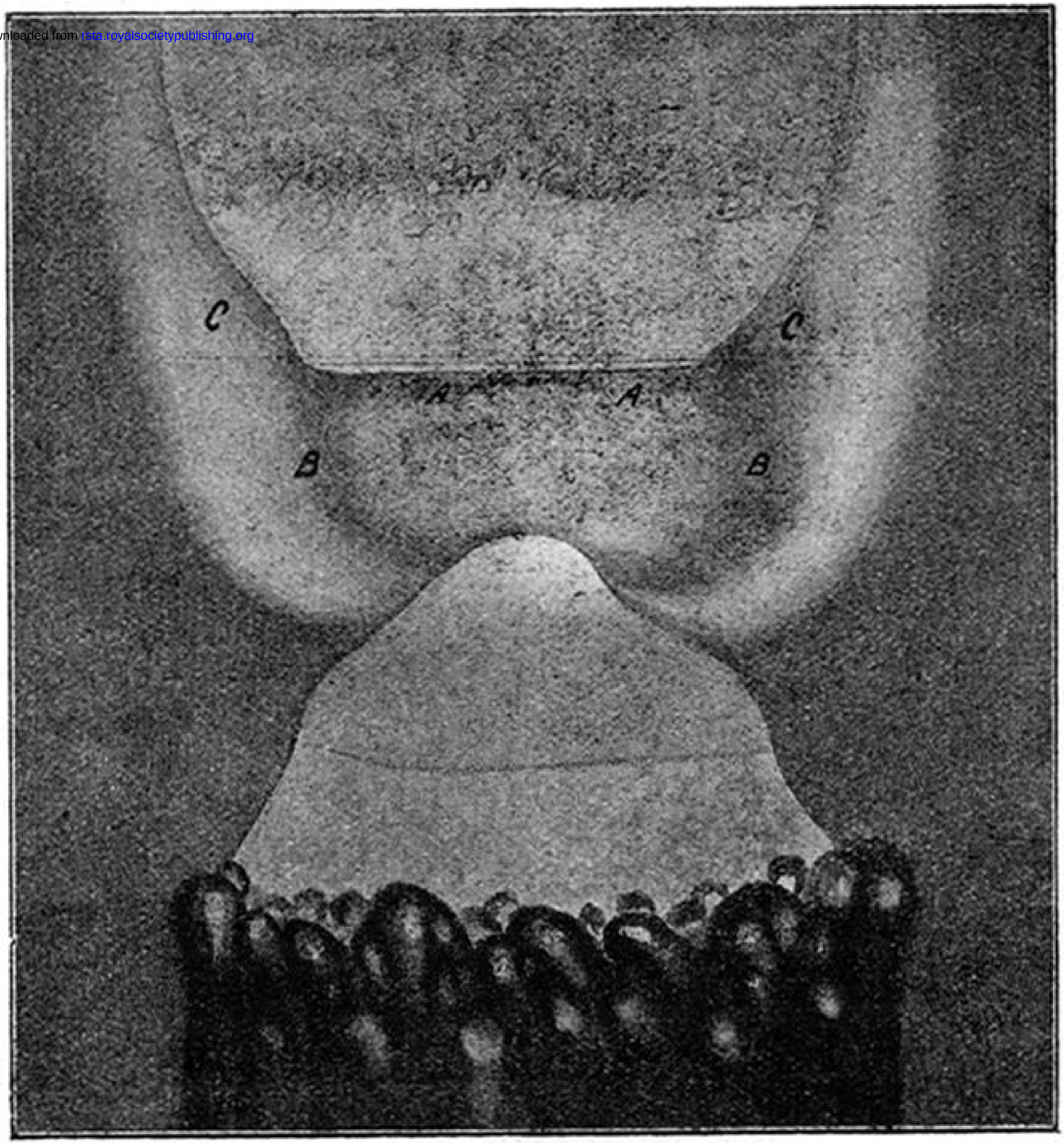

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ig. 1. Enlarged image of arc and carbons with positive carbon on top. *AA*, purple mist, *BB*, shadow, *CC*, green flame.